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CORRELATION PROPERTIES OF DIFFERENTIAL REFLECTIVITY AND THEIR IMPLICATIONS FOR RADAR METEOROLOGY

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V.N. Bringi
T.A. Seliga

The Ohio State University Research Foundation
1314 Kinnear Road
Columbus, Ohio 43212

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This final report deals with the theoretical and experimental statistical properties of differential reflectivity, or Z_{DR}. The latter are based on the time series radar data obtained with the high resolution, "fast" polarization switching Chilbolton radar system located in Southern England and operated by the Rutherford and Appleton Laboratories. An optimum estimator of Z_{DR} is derived and comparisons of theoretically and experimentally obtained standard errors show excellent agreement. An examination of the effects of noise on the accuracy of Z_{DR} measurements shows that signal-to-noise ratio of 20dB or greater is

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) required to achieve high accuracy estimate of Z_{DR}. Time series radar data from England are used to determine the fluctuation spectrum (periodogram) of Z_{DR} for the purpose of searching for signatures characteristic of polarization dependent backscattering processes such as drop vibration, canting, etc. A theoretical model of raindrops vibrating at their natural oscillation frequency (with no motion) is developed and the periodogram of Z_{DR} is computed for comparison with measured data.

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1. Introduction

It is well known that dual-measurement techniques are needed for accurate estimation of rainfall rate using radar.[1] Of the several dual-measurement schemes postulated in the literature, the technique based on differential reflectivity, or Z_{DR}, appears most promising.[2,3,4,5] This final report deals with the theoretical and experimental statistical properties of Z_{DR}. The latter are based on the time series radar data obtained with the high resolution, "fast" polarization switching Chilbolton radar system located in Southern England and operated by the Rutherford and Appleton Laboratories.[6]

The major results of the research are:

A. The optimum estimator of Z_{DR} was derived and excellent agreement was obtained between theoretically and experimentally obtained standard errors.[7] Using pulse-to-pulse polarization switching with a prf of 610 Hz, Z_{DR} defined as $10 \log (Z_H/Z_V)$ can be measured with standard errors in the range 0.1 - 0.2dB at a single range gate using about 40 - 60 independent samples. This high accuracy is achieved due to the high degree of cross-correlation between Z_H and Z_V, estimated to have median values of 0.97 or higher.

B. The effects of noise on the accuracy of Z_{DR} measurements was examined. A signal-to-noise ratio of 20dB or greater is required to achieve high accuracy estimates of Z_{DR}.

C. The time series radar data from England was used to determine the fluctuation spectrum (periodogram) of Z_{DR} for the purpose of searching for signatures characteristic of polarization dependent backscattering processes such as drop vibration, canting, etc. A theoretical model of raindrops vibrating at their natural oscillation frequency (with no motion) was developed and the periodogram of Z_{DR} was computed for comparison with measured data.

D. Earlier, the authors had suggested using the differential propagation phase shift in rain between the horizontal and vertical polarizations as another parameter to be combined with Z_{DR} for accurate rainfall measurement.[8] The accuracy of this differential phase shift measurement was estimated as a function of the cross-correlation parameter and the number of independent samples in a manner similar to that used for the Z_{DR} analysis.

This final report summarizes the results (outlined above) obtained during the contract period. All the tasks set forth in the original work statement have been satisfactorily completed.

2. Statistical Model

A scattering model will be considered for which the co-polar backscattered signal, for a horizontally or vertically polarized transmitted pulse, can be written in the form

$$A_{H,V} \exp(i\phi_{H,V}) = \sum_{j=1}^{m} F_{j(H,V)} \exp(i\theta_{j(H,V)}) \qquad [1]$$

where $A_{H,V}$ and $\phi_{H,V}$ are the resultant amplitude and phase, and $F_{j(H,V)}$ and $\theta_{j(H,V)}$ are the backscattered amplitudes and phases from the jth scatterer in the scattering (pulse) volume. The subscripts (H,V) refer to horizontal or vertical polarization, respectively.

We resolve $A_{H,V}$ into their phasor components (X_H, Y_H) and

(X_V, Y_V) where $A_{H,V} = (X_{H,V}^2 + Y_{H,V}^2)^{1/2}$ and assume that the set of random variables (X_H, Y_H, X_V, Y_V) are normally distributed with zero mean and covariance matrix given by:

$$\bar{\bar{\Sigma}} = \begin{bmatrix} \sigma_H{}^2 & 0 & \rho\sigma_H\sigma_V & 0 \\ 0 & \sigma_H{}^2 & 0 & \rho\sigma_H\sigma_V \\ \rho\sigma_H\sigma_V & 0 & \sigma_V{}^2 & 0 \\ 0 & \rho\sigma_H\sigma_V & 0 & \sigma_V{}^2 \end{bmatrix} \qquad [2]$$

where $E[X_{H,V}^2] = E[Y_{H,V}^2] = \sigma_{H,V}^2$ and $E(X_H X_V) = E(Y_H Y_V) = \rho\sigma_H\sigma_V$.

2.1. Optimum Estimator of Z_{DR}

This model is valid for coincident sampling of the pulse volume at the two polarizations (H, V). Z_{DR} and Z_{DR}(dB) are defined as $\sigma_H{}^2/\sigma_V{}^2$ and $10 \log (\sigma_H{}^2/\sigma_V{}^2)$, respectively. The optimum estimator of Z_{DR} is of the form

$$\hat{Z}_{DR} = \left(\frac{1}{m} \sum_{i=1}^{m} A_{H_i}^2\right) \div \left(\frac{1}{m} \sum_{i=1}^{m} A_{V_i}^2\right) \qquad [3]$$

where A_{H_i,V_i} form a pair-wise set of identically distributed (Eq. [2]) random samples. This estimator is termed the square law estimator and the calculated standard error (dB) as a function of sample size, m, and cross-correlation coefficient ρ is shown in Fig. 1. Note the advantage of having ρ as high as possible in order to reduce the standard error of Z_{DR}(dB) to 0.1 - 0.2dB with 40 - 60 independent samples.

2.2. Effects of Noise

The effects of white, Gaussian noise can be simulated by altering the covariance matrix as defined in Eq. [2] to $\bar{\bar{\Sigma}} + \bar{\bar{I}}\sigma_n{}^2$ where $\bar{\bar{I}}$ is the

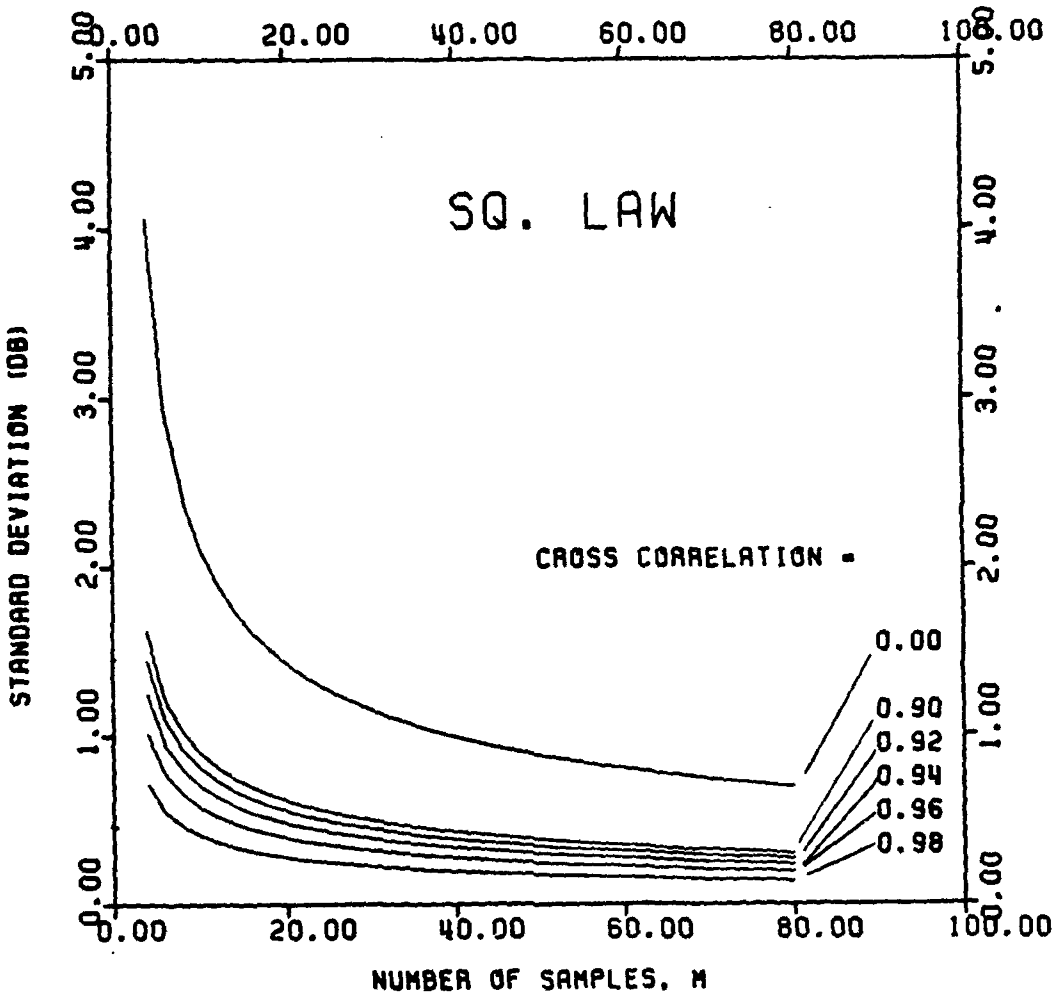

Figure. 1. Standard deviation (dB) of the square law estimator as a function of sample size m and cross-correlation coefficient ρ.

identity matrix and σ_n^2 is the noise power, assumed equal at the two polarizations. The effect of noise results in a reduction of ρ to ρ^* defined as

$$\rho^* = \frac{\rho}{\left(1 + \frac{1}{SNR}\right)^{\frac{1}{2}}\left(1 + \frac{Z_{DR}}{SNR}\right)^{\frac{1}{2}}} \qquad [4]$$

where the signal-to-noise ratio, $SNR = \sigma_H^2/\sigma_n^2$ and $Z_{DR} = \sigma_H^2/\sigma_V^2$. It is estimated from Eq. [4] that an SNR > 20dB is required to maintain the high accuracy in the measurement of Z_{DR}.

2.3. Differential Phase Shift

The potential use of the differential phase shift in rain due to propagation depends to a large degree on the possible measurement accuracy that can be achieved for this parameter. The probability density function of $\Delta = \phi_H - \phi_V$ is given by[9]

$$f(\Delta) = \frac{(1-\rho^2)\left[(1-\rho^2\cos^2\Delta)^{\frac{1}{2}} + \rho\cos\Delta\left\{\pi - \cos^{-1}(\rho\cos\Delta)\right\}\right]}{2\pi(1-\rho^2\cos^2\Delta)^{3/2}} \qquad [5]$$

where $\phi_{H,V}$ is defined by Eq. [1] and the covariance matrix $\bar{\bar{\Sigma}}$ is defined in Eq. [2]. We assume phase difference measurements Δ are available at two range gates separated by a distance ℓ so that the mean propagation differential phase shift is given by $Re(k_H - k_V)2\ell$ where $k_{H,V}$ is the propagation constant at the two polarizations. Neglecting the differential phase shift due to backscatter at the long wavelength (say 10 cm), the fractional standard deviation (FSD) of the differential phase shift was determined to be

$$FSD = \frac{\text{standard deviation}}{\text{mean}} = \frac{1}{2\ell \mathrm{Re}(k_H - k_V)} \sqrt{\frac{2}{m}} \left\{ \int_{-\pi}^{\pi} \Delta^2 f(\Delta) d\Delta \right\}^{\frac{1}{2}} \quad [6]$$

In Fig. 2, we show plots of m versus ρ for a constant FSD of 0.1 and 0.2 assuming a rainfall rate of 100 mm hr^{-1} over a path length of 2ℓ = 2km. At lower rainfall rates, the number of samples, m, increases very rapidly if FSD's of 0.1 or 0.2 are to be maintained. The effects of noise can be determined by replacing ρ by ρ^* as defined in Eq. [4].

3. Radar Measurements

The measurements used to obtain the statistics of Z_{DR} were made using a 10 cm wavelength radar on a fully-steerable 25 m diameter antenna in Southern England. The essential characteristics of the radar system are given in Table 1. The antenna was fed from its prime focus by a circular scalar feed, so as to best match the aperture of the dish and present the same beam pattern for vertical and horizontal polarizations. A fast polarization switch coupled this feed to the radar in such a way that alternate (transmitted) radar pulses were vertically and horizontally polarized. The received power was always copolar. Because of the high radar power and the rapid switching rate, a mechanical rotating chopping disc was used rather than an electrical switch. The radar PRF (600 Hz) was generated from the rotating disc (6000 rpm) to ensure synchronization.

The received radar power for each of the polarizations was then processed through the same receiver, IF log amplifier, and analog to digital converter. Measurements were made only with the antenna

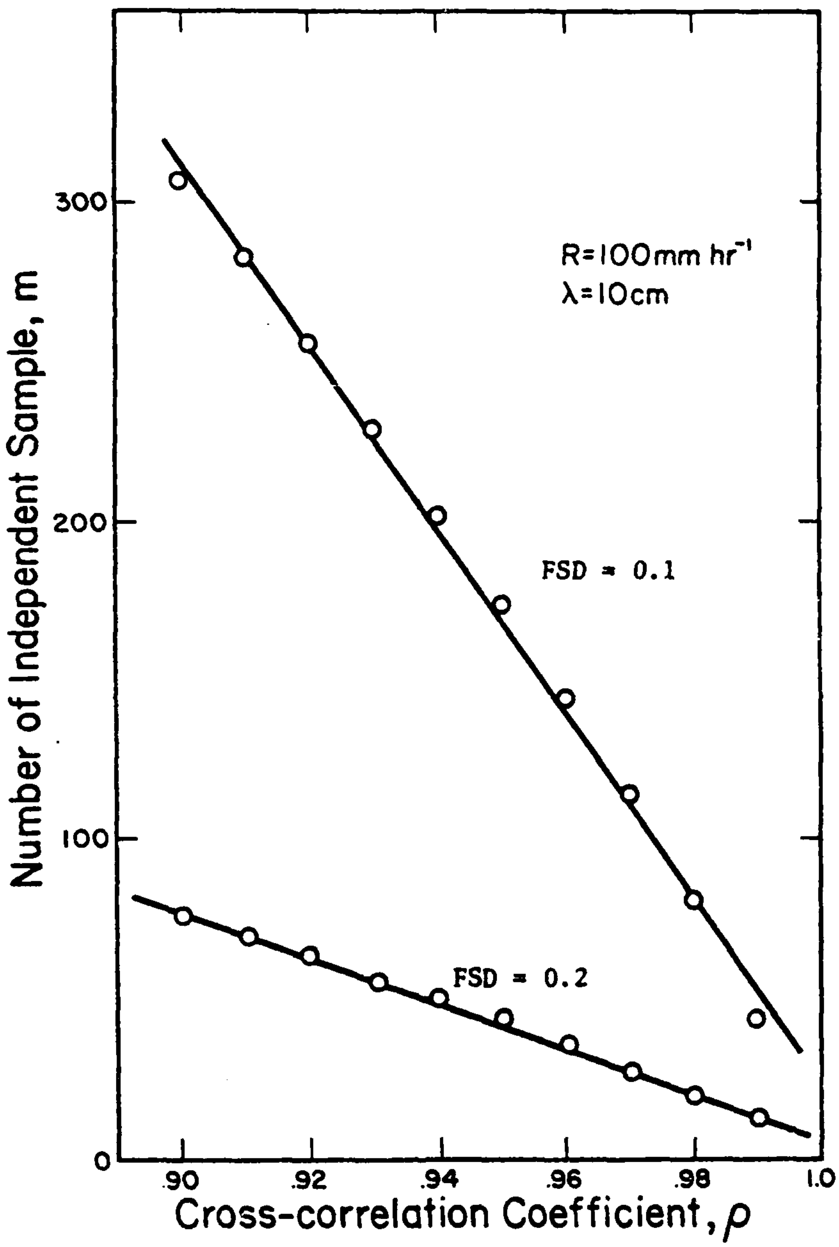

Figure 2. Number of independent samples, m, versus cross-correlation coefficient ρ to determine mean differential phase shifts with Fractional Standard Deviations (FSD) of 0.1 and 0.2. Rainfall rate is 100 mm hr^{-1}, and wavelength of 10 cm is assumed.

TABLE 1

RADAR CHARACTERISTICS

Frequency	:	3076 MHz
Peak power	:	500 kw
Pulse width	:	0.5 μs
Pulse repetition frequency	:	610 Hz
Polarization sampling	:	alternate vertical and horizontal
Antenna measured gain	:	53 dB
Antenna 3dB beamwidth	:	16 arcminutes
IF bandwidth	:	10 MHz
Digitization range	:	180 steps of 0.3dB
Range gate width	:	0.3 μs

pointing fixed, and with a single range gate. Two-hundred 8-bit samples from each polarization were stored together on magnetic tape as a data record, corresponding to a two-channel stream lasting 640 ms (see Fig. 3). 860 ms were required between records for data transfer and reset conditions. Some 3,500 records of this type were available for analysis. Some of the data records were obtained for radar pulse volume return from near the center of raincells, others from the edge of raincells, and yet others from regions of the melting layer.

4. Data Analysis

The first step in the analysis of the data was to conduct a goodness-of-fit test to the distribution $F(u) = \int_0^u f(x)dx$, where $u = \frac{A_H/\sigma_H}{A_V/\sigma_V}$, and the probability density function f(u) is given as[7]

$$f(u) = \frac{2u(1 + u^2)(1 - \rho^2)}{[(1 + u^2)^2 - 4\rho^2 u^2]^{3/2}} \qquad [7]$$

Note that the above function is based on a single, coincident sample of $A_{H,V}$ from a given range gate. The Chi Square test was performed to see if the measured data originated from the null distribution, $F_0(u)$. Approximately 1,500 records were analyzed and about 90% were accepted at a 1% significance level. This high acceptance rate established both the high quality of the radar data and the adequacy of the statistical model.

The data was next analyzed to estimate the standard deviation, σ, of $\hat{Z}_{DR}$(dB) using the optimum square law estimator defined in Eq. [3]. The median value of σ based on 125 data sets was found to be 0.214dB which compares very well with theoretical estimates in the range 0.16 - 0.32dB, using ρ in the range 0.92 - 0.98. Further range or time

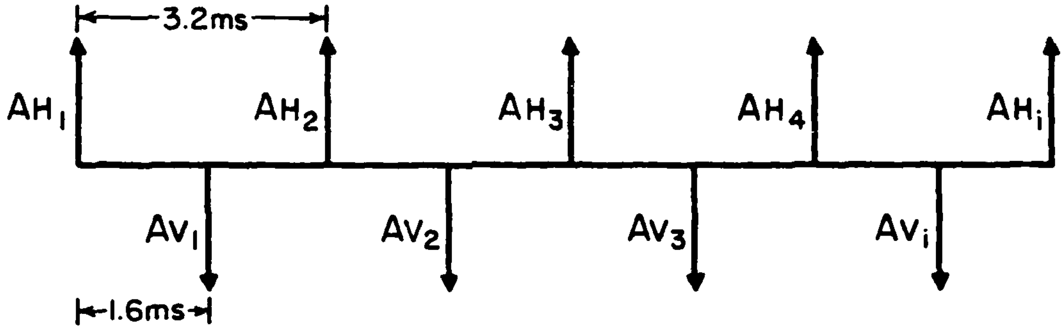

Figure 3. One record of time series data from a fixed range gate lasting 640 msec.

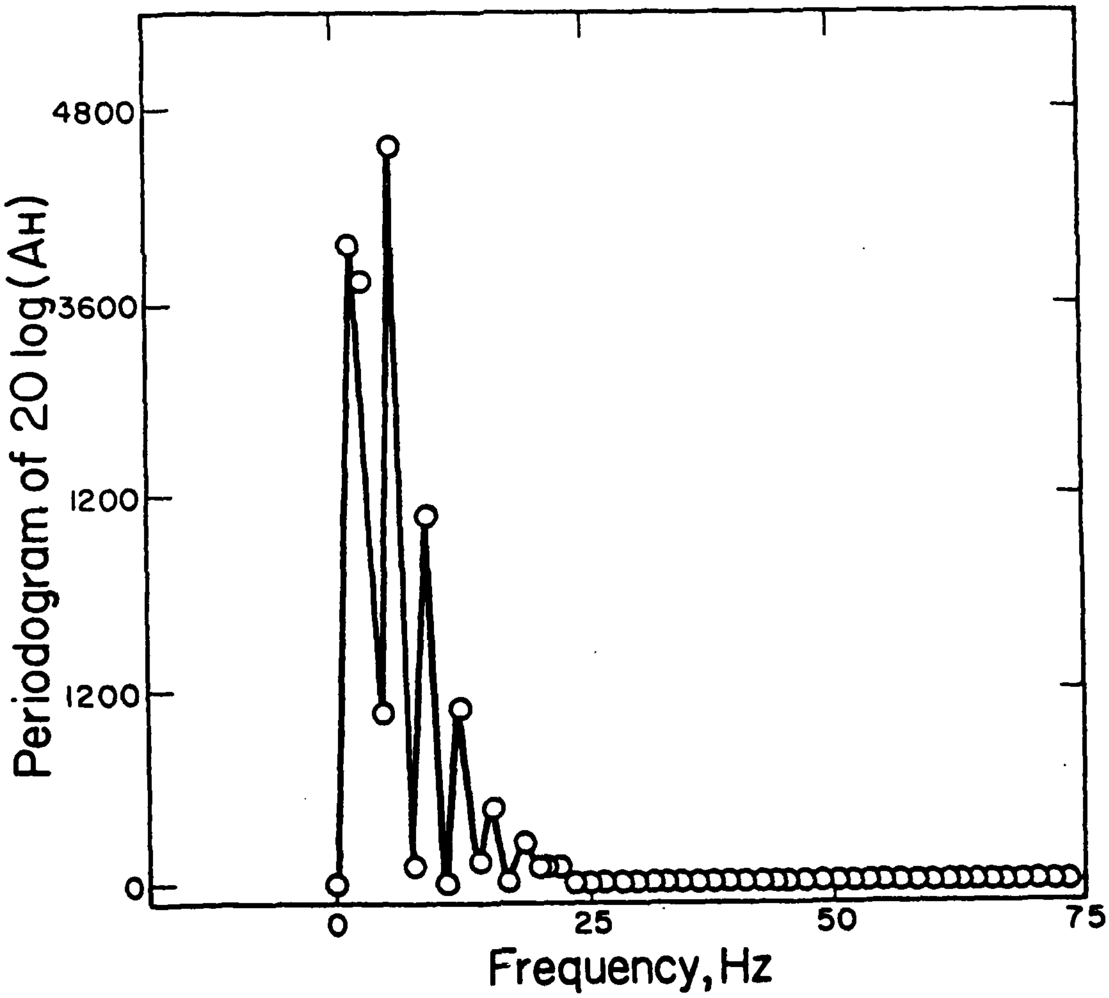

Figure 4. Periodogram of 20 log (A_H) with adjusted mean based on 200 samples and a time duration of 640 msec.

averaging is expected to reduce σ to around 0.05dB. The same 125 data sets yielded a median value of $\rho = 0.97$. Additional details of the preceding data analysis are given by Bringi et al.[7]

The measured data is based on a time lag of 1.6 msec between the horizontally and vertically polarized signals, while the theory assumes coincident sampling at the two polarizations. It is expected that coincident sampling would produce higher cross-correlation, thus further reducing the standard error in the measurement of Z_{DR}. However, system complexity would increase due to a two-channel receiving system with more stringent antenna and feed requirements. Decorrelation between A_H and A_V for sequential sampling can arise due to two causes, viz., (a) drop vibration, canting, etc. and (b) the Doppler spectrum. In severe, convective storms the decorrelation due to the Doppler spectrum would dominate causing a significant reduction in $\rho(\sim 0.90)$, while for a narrow Doppler spectrum the decorrelation would be slight due to drop vibration, canting, etc. As a result, more "independent" pairwise samples of $Z_{H,V}$ will occur due to the combination of the above effects. Nevertheless, sequential polarization sampling is preferred due to its simplicity and adaptability of existing single polarization radar systems to possible future modification.

5. Spectrum Calculations

The time series radar data obtained from measurements was used to estimate the fluctuating spectrum of $20 \log A_H$ and $20 \log (A_H/A_V)$. A typical periodogram of $20 \log A_H$ is shown in Fig. 4 with adjusted mean using a standard procedure called SPECTRA.[10] Almost all of the data records show a similar narrow spectrum with estimated standard deviation of about 0.5 $msec^{-1}$ at 10 cm wavelength.

Sample periodograms of $20 \log (A_H/A_V)$ are shown in Fig. 5 and 6 with adjusted means. The spectrum of Fig. 5 is typical of records which have very low mean Z_{DR}'s ($\leq$ 0.25dB). These spectra are broad as compared to those having higher mean Z_{DR}'s ($\approx$1 - 1.5dB) of which a sample is shown in Fig. 6. Unfortunately, no data having mean Z_{DR} values in the range 2 - 5dB, as typically found in rain in the continental United States, were available. It, therefore, would be desirable to analyze data from a variety of rain conditions and at a larger number of range gates before drawing any firm conclusions regarding the meteorological significance of the Z_{DR} spectrum.

5.1. Fluctuation Model

It is well known that raindrops vibrate at their natural frequency while falling at their terminal velocity. This vibration causes small fluctuations of Z_{DR} about a mean value and it is of interest to calculate the fluctuation spectrum based on a simple model with no motion. An exponential drop size distribution was assumed with each drop oscillating at a natural frequency given by $f = 4.22a_o^{-1.47}$ where a_o is the radius of the equivolumic spherical drop. The drops were assumed to oscillate sinusoidally between spherical and oblate spheroidal shapes with maximum amplitudes of 20% of the equivolumic spherical diameter. Gans' scattering theory was used to calculate $Z_{DR}(t)$. The resultant periodogram with adjusted mean is shown in Fig. 7 for a mean Z_{DR} of 1.3dB. Even though the fluctuations of Z_{DR} about its mean value is very small, the comparison between Fig. 7 and the actual periodogram of Z_{DR} as shown in Fig. 6 is striking. If the Doppler spectrum has a large variance, the Z_{DR}

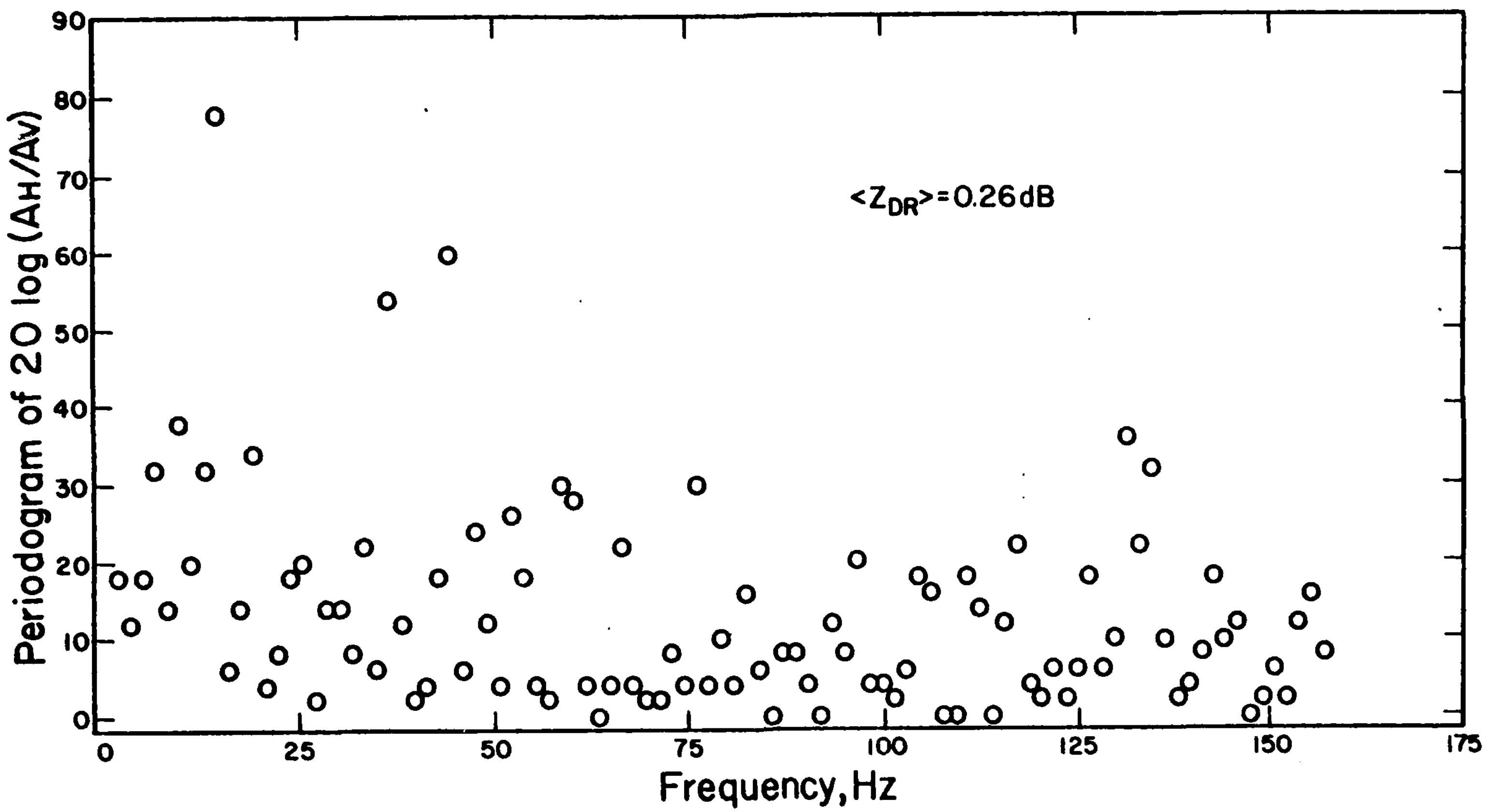

Figure 5. Periodogram of 20 log (A_H/A_V) with adjusted mean based on 200 samples and a time duration of 640 msec.

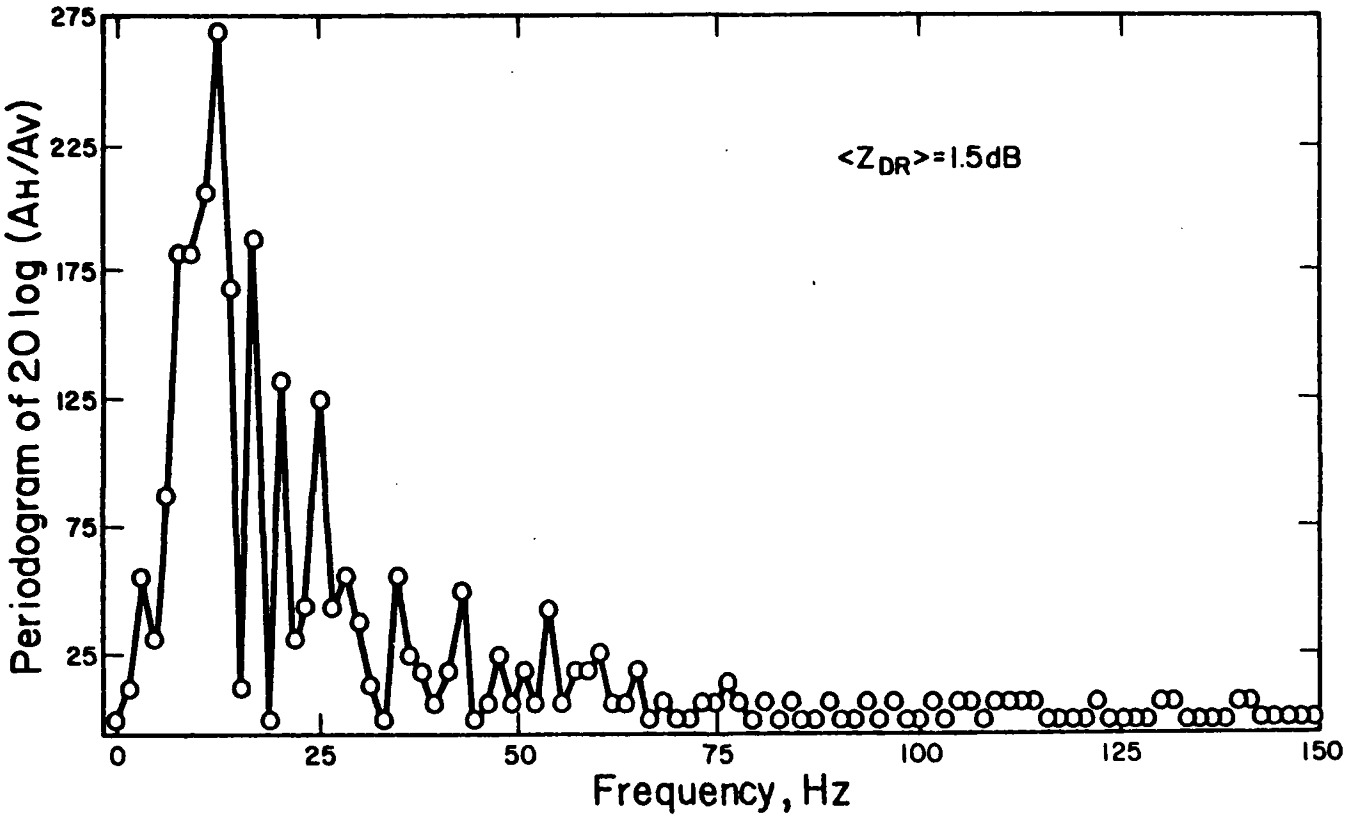

Figure 6. Periodogram of 20 log (A_H/A_V) with adjusted mean based on 200 samples and a time duration of 640 msec.

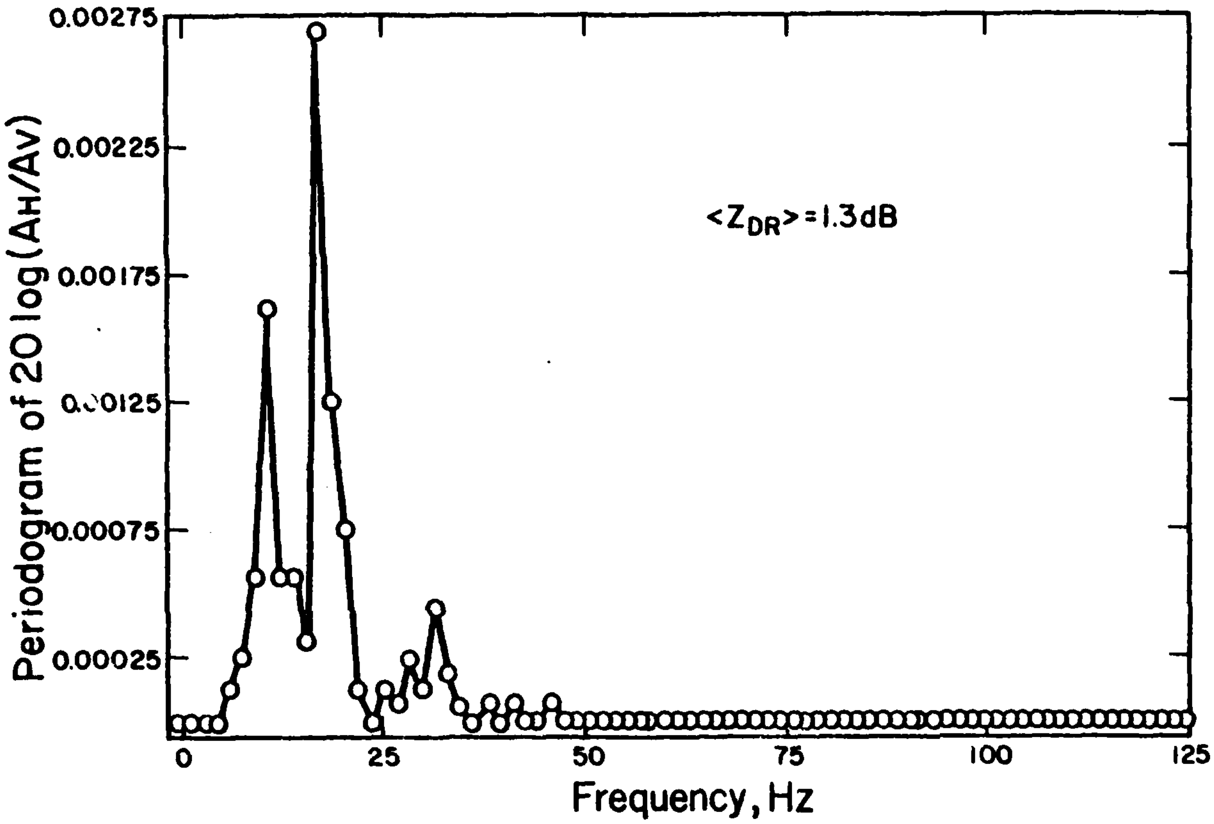

Figure 7. Computed periodogram of 20 log (A_H/A_V) with adjusted mean based on a raindrop vibration model.

spectrum is not expected to yield any information on raindrop vibration. Other methods, such as broad band noise techniques or coherent polarization radar techniques, may be required to obtain further information about the hydrometeors.[11,12] Also, higher resolution recording of the co-polar reflectivities is desirable in order to retain all of the important spectral characteristics of Z_{DR}.

6. Conclusions

The differential reflectivity parameter, Z_{DR}, shows great promise as a valuable parameter in radar meteorology. It can be measured accurately and rapidly, as shown by the statistical characteristics of the Z_{DR} signal when compared with actual radar data obtained from the Chilbolton radar system. The optimum estimator of Z_{DR} was described, and the sequential method of polarization sampling is recommended, provided sampling occurs within the decorrelation time of the scatterers in the pulse volume. It is shown that Z_{DR} can be measured with a standard error of 0.2dB at a single range gate using 40 - 60 independent samples in a time duration of 1 sec. Furthermore, the radar data is shown to be consistent with a statistical model describing the backscattering from the pulse volume. The effects of noise in the model were also considered, and the same model was used to predict the accuracy to which differential propagation phase shifts could be measured.

The time series data obtained from the Chilbolton radar was analyzed spectrally for both the horizontal amplitude returns and the Z_{DR} signal. Large differences in the shape of the spectrum were found for conditions of small (~0.25dB) and large (1.5dB) mean Z_{DR}'s. The fluctuation spectrum of the horizontally polarized signal was very narrow with an estimated Doppler standard deviation of 0.5 $msec^{-1}$ at 10 cm wavelength for all the measured data. The Z_{DR} spectrum obtained with real data was compared with the expected Z_{DR} spectrum from a model, exponential rain drop size distribution where the drops are fixed in space and vibrate at their natural oscillation frequency. Although the Z_{DR} fluctuations in the model are very small, the shapes of the model spectra and the real spectra (for mean $Z_{DR} \approx 1.5$dB) were similar. It is recommended that further time series data be analyzed under conditions of larger mean Z_{DR} and at a larger number of range gates.

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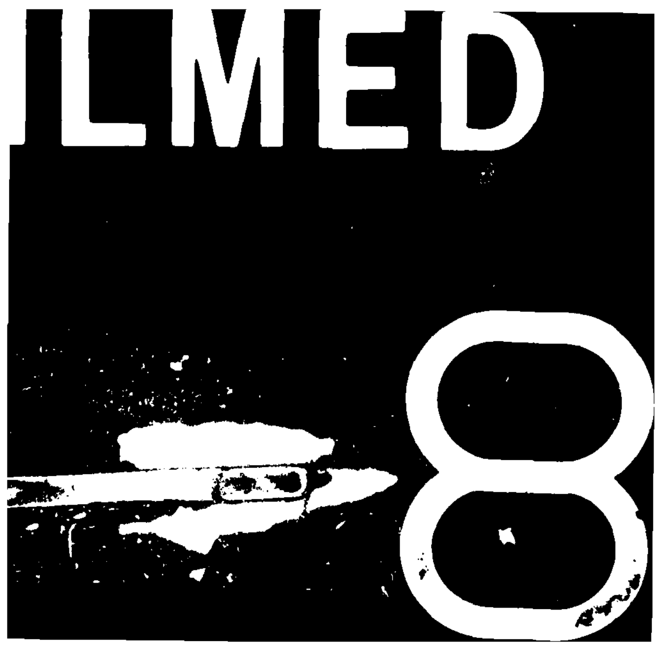
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